Honours Topics

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⚀ Random walk on a quadrant

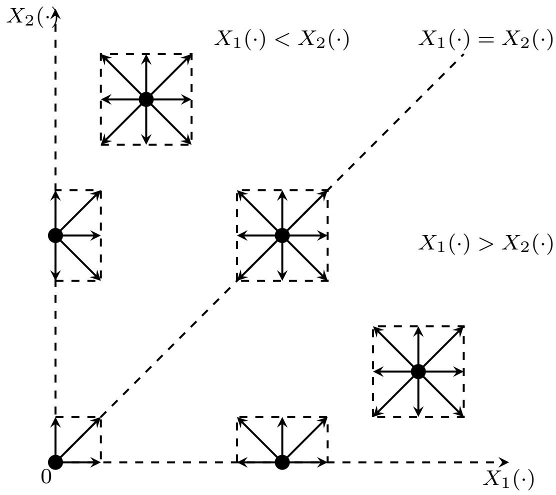

Figure 1: Neighbourhood random walk on a quadrant is a process in which you start at some point $(X_1(0), X_2(0))$ in phase $\varphi(0)$ and then move randomly. Phase $\varphi(t)$ of a continuous-time Markov chain modulates times spent in each position $(X_1(t), X_2(t))$ and directions of movement at each jump of a process (to a different position, or phase, or both).

This process has applications to a wide range of real-life systems in many areas (e.g. tandem network, population with two types of individuals, movement of species on a map, evolution of gene families, spread of epdiemics etc.)

A challenge has been the analysis and computation of long-run and time-dependent quantities, due to the dimension of the process, and the boundary condition, $X_1(t) \geq 0$, $X_2(t)) \geq 0$, which makes the analysis harder. Thankfully, new ideas have emerged in "*Random walk on a quadrant: mapping to a one-dimensional level-dependent Quasi-Birth-and-Death process (LD-QBD)*" M.M. O'Reilly, Z. Palmowski, A. Aksamit (2022).

The aim of this project is to construct numerical examples for simulation and application of the random walk, based on these new ideas. This project will suit a student with interest in stochastic modelling. □

⚁ Applications of QBDs to modelling patient flow

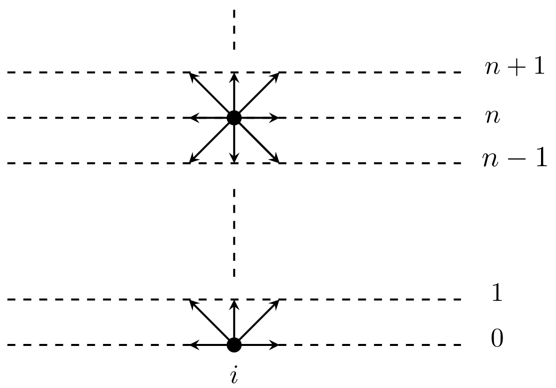

Figure 2: We model patient flow in a hospital as a quasi–birth–and–death process (QBD) where the total number of patients $X(t) = n$ changes according to some information $\varphi(t) = i$ about the system. That is, $\varphi(t)$ drives the evolution of the system. As example, the variable $\varphi(t)$ may be the total number of complex patients in the system, who require more resources and spend more time in the system.

Using this QBD, we may compute multiple useful metrics e.g. the distribution of time until the number of patients drops below a certain threshold, or the probability of a hospital going above a threshold within a given time interval.

This project builds on the research of the multidisciplinary team *Designs for a Better Patient Journey* in collaboration with the Royal Hobart Hospital and Tasmanian Department of Health. Thanks to obtaining data of patient flow from the past five years, we can fit QBD parameters to data and perform analysis.

The aim of this project is to fit parameters of different QBD models to this data and perform analysis to gain insights about patient flow. This project will suit a student with interest in operations research. □

Selected examples of past topics:

Modelling Hospital Escalation Levels as Quasi-Birth-and-Death Processes

(Mr Augustus Grant, Honours, 2021)

Summary: Hospitals are consistently close to capacity and beholden to stochastic fluctuations in demand, particularly from the Emergency Department. In an attempt to control these fluctuations, many hospitals use a system of escalation levels to determine policy in relation to who as admitted to hospital and when healthy patients are discharged. This approach is thought to reduce strain on a hospital, reduce the risk and severity of overcrowding and allow a hospital to continue running in optimal conditions, even as demand varies. However, there is a lack of scientific tools supporting this complex process, and so these decisions are made manually, based on the intuition and experience of hospital management. We present a number of classes of models, designed to inform these decisions. To do this, we model the hospital as a quasi–birth–and–death process (QBD), which essentially is a continuous-time Markov chain with a two-dimensional state (n, i) such that n records the number of patients in the system, and i records some additional information such as the number of complex patients in the system.

Using this QBD, we compute several useful metrics for informing decisions within hospitals, including the distribution of time until the number of patients drops below a certain threshold, or the probability of a hospital going above a threshold in a given time interval. The choice of both models and metrics considered are based on recommendations by a multidisciplinary team of healthcare system designers and managers, doctors and mathematicians. Our findings can be used directly by this team to inform hospital policy and improve the future running of the hospital. This work contributes to the multidisciplinary team 'Designs for a Better Patient Journey'.

Modelling of optimal decision making in healthcare systems

(Mr Sebastian Krasnicki, Honours, 2020)

Summary: When a patient arrives to a hospital, a decision must be made as to which ward, room and bed that patient is assigned to. As hospitals are highly stochastic in nature, optimal assignments of patients may be unclear. In this thesis, we model the hospital system as a Markov decision process and present two models of patient assignment; the Ad-Hoc planner for assigning a single patient at a time, and a novel model for assigning multiple patients at a time, the 24-hour planner. Work on this paper is being done in conjunction with healthcare experts from the University of Tasmania's School of Medicine and the Health Planning Unit of the Tasmanian Department of Health, to ensure that the model captures realistic hospital behaviour.

For each of the models we present, we modify established approximate dynamic programming techniques, and perform numerical analysis to show that these methods find near-optimal patient assignments in terms of both long-run cost and hospital congestion. We identify key features of the hospital that produce the most stable estimates of optimal patient assignments. Finally, we discuss how these results will be used to explore more complex methodologies.

Stochastic Fluid Models and their Applications

(Mr Adrian Tanner, Honours, 2018)

Summary: A wide range of real-life systems with engineering, finance and environmental significance are stochastic in nature. Since these systems are important to society, there is interest in modelling them so as to make predictions of how they will behave in the future. Stochastic Fluid Models (SFMs) are a class of models with a two-dimensional state-space $\{\varphi(t), X(t)\}$ consisting of a phase $\varphi(t) \in \mathcal{S}$ and a level $X(t) \in \mathbb{R}$. The phase variable $\varphi(t)$ is often used to describe the state of some physical environment that we wish to model, such as on/off mode of a switch in a telecommunications buffer, peak/off-peak period in a telephone network, or wet/dry season in reservoir modeling. The level variable $X(t)$ is used to model some continuous performance measure of the system, eg the amount of data in a buffer, the level of water in a reservoir, or the revenue earned by time t. The model assumes that the transitions between phases occur according to some underlying continuous-time Markov Chain (CTMC). Furthermore, the rate $c_i = dX(t)/dt$ of increase of the fluid level at time t depends on the phase $\varphi(t) = i$ at time t, and so the Markov Chain is the process that *drives* the fluid level at time t.

The aim of the project was to review the literature in the area of applications of SFMs and construct numerical examples using efficient algorithms and simulations. We developed a SFM to model real life systems that deteriorate over time and are inspected at fixed time intervals. If the level of deterioration exceeds some intervention level then the system is replaced, and applied it to optimise the inspection interval, in terms of cost and risk, for wooden power poles under different maintenance strategies.

Markov Models for Microsatellite Mutation
(Mr Tristan Stark, Honours, 2013)

Summary: Markov Chains is the most important class of models in Probability Theory, due to their modeling potential and numerical tractability. The aim of this project was to review the literature concerning the application of Markov Chains in Phylogenetics, the study of evolutionary relation among groups of organisms (e.g. species, populations), for example see:

Calabrese, P., Sainudiin, R., Models of Microsatellite Evolution, in Statistical Methods in Molecular Evolution, R. Nielsen, Editor. 2005, Springer, p. 289-306.

The project focused on understanding these models, particularly where they appear as continuous-time Markov chains, as well as investigating the strengths of individual models, and outlining the inherent assumptions of the current modelling paradigm. We presented a novel model with the intention to motivate the investigation of modelsin its class. This model was a foundation of ideas that were later explored in a PhD project by Dr Tristan Stark.

Stochastic Fluid Model for Deteriorating Systems
(Mr Andrew Haigh, Honours, 2012)

Summary: Markovian-modulated models are a class of models with a two-dimensional state-space consisting of a phase and a level. The phase variable is often used to describe the state of some physical environment that we want to model. Simple two-phase examples are on/off mode of a switch in a telecommunications buffer, peak/off-peak period in a telephone network, or wet/dry season in reservoir modeling. The model assumes that the transitions between phases occur according to some underlying continuous-time Markov Chain. Furthermore, the rate of increase of the fluid level at time t depends on the phase at time t, and so the Markov Chain is the process that drives the fluid level at time t. The aim of the project was to explore and review the current literature in the area.

We described the construction of a new class of stochastic fluid models, the deteriorating system SFM, that can be used to model systems that deteriorate over time and have to be replaced when they fail. This model has a novel combination of behaviours including a finite number of fluid layers, within each of which the phase may have a different behaviour, and the possibility of the fluid level jumping instantaneously from the upper boundary to 0, to model the system's replacement. The most important performance measures of the model, the distribution of the length of the lifetime and the stationary (long-run) distribution, are derived. We describe how these performance measures can be calculated efficiently.

The results of this work were later published in:

Stochastic model for maintenance in continuously deteriorating systems. Samuelson A., Haigh A., O'Reilly M.M., Bean N.G. (2017) European Journal of Operational Research, 259(3), pp. 1169-1179.